Joint tests for regularity and autocorrelation in allocation systems

Deschamps, P.J.

Publication date:
1994

Document Version
Publisher's PDF, also known as Version of record

CentER for Economic Research

REPRINT

Joint Tests for Regularity and Autocorrelation in Allocation Systems

by
P.J. Deschamps

Reprinted from Journal of Applied Econometrics,
Vol. 8, 1993

Reprint Series
no. 155

TILBURG UNIVERSITY

Address : P.O. Box 90153, 5000 LE Tilburg, The Netherlands
Phone : +31 13 663102
Telex : 52426 kub nl
Telefax : +31 13 663066
E-mail : center@kub.nl

ISSN 0924-7874

1994

CentER
for
Economic Research

REPRINT

Joint Tests for Regularity and Autocorrelation in Allocation Systems

by
P.J. Deschamps

Reprinted from Journal of Applied Econometrics,
Vol. 8, 1993

Reprint Series
no. 155

JOURNAL OF APPLIED ECONOMETRICS, VOL. 8, 195–211 (1993)

JOINT TESTS FOR REGULARITY AND AUTOCORRELATION IN ALLOCATION SYSTEMS

P. J. DESCHAMPS
Université de Fribourg, IAUF, Miséricorde, CH-1700 Fribourg, Switzerland

SUMMARY

In the context of allocation models with vector autoregressive errors we propose a convenient procedure, based on the Lagrange multiplier principle, for testing any possible combination of absence of serial correlation, homogeneity, and symmetry against any possible alternative which specifies autocorrelation of an arbitrary given order. We also derive generic expressions for the maximum likelihood estimation of the models under six possible combinations of constraints. The methodology is illustrated with the Rotterdam model and the differential AIDS model, both estimated from the same quarterly British data.

1. INTRODUCTION

The pioneering work of Berndt and Savin (1975) and Lau (1978) made the profession aware that the adding-up condition has important consequences for the specification of dynamic error processes in allocation systems (multivariate linear regression models with singular error covariance matrix). Berndt and Savin recognized that the only vector autoregressive processes compatible with adding-up are those in which the matrices R_j^* of autocorrelation coefficients satisfy $\iota' R_j^* = \rho_j \iota'$, where ι is a column vector of ones and ρ_j is an unknown constant. This condition was later proved by Lau (1978) to be necessary and sufficient for adding-up to hold in an autocorrelated allocation system.

Berndt and Savin (1975) also present a procedure, based on the work of Hendry (1971), for the maximum likelihood (ML) estimation of autocorrelated allocation systems. As usual, this procedure involves the deletion of an equation; the estimates are invariant with respect to the index of the deleted equation. They also show that the matrices R_j^* of autocorrelation coefficients are not identifiable without further restrictions. This does not present a problem insofar as the primary parameters of interest are the coefficients of the observable variables (e.g. price and income coefficients) rather than the elements of R_j^*.

One implication of the Berndt–Savin results is the inadequacy of those tests for autocorrelation that are based on the residuals of a single equation, such as the Durbin–Watson (DW) statistic (the single-equation DW test has, however, been used in the empirical literature on allocation systems; see e.g. Deaton and Muellbauer, 1980a). The correct procedure involves jointly testing for autocorrelation in the full system, taking into account the restrictions on R_j^* implied by adding-up. If one is willing to assume that the matrices R_j^* are diagonal, adding-up implies $R_j^* = \rho_j I$. Since the autocorrelation coefficients are then identical across all equations, it would appear reasonable (though not rigorous) to compute a single DW statistic from the pooled residuals of the entire system. Usually, however, the model attempts to explain the demand for heterogeneous commodities (e.g. food and housing); in this

0883–7252/93/020195–17$13.50

Received December 1991
Revised October 1992

instance an assumption of equal correlation coefficients across commodities is clearly unattractive. It is then much more appealing to test a model with spherical disturbances against one where the matrices R_j^* are not restricted in any way. One may for this purpose use the likelihood ratio (LR) test, in the fashion of Berndt and Savin; or the more recent score or Lagrange multiplier (LM) test statistic initially proposed by Rao (1948) and Aitchison and Silvey (1960), and later investigated by Godfrey (1978) and Breusch and Pagan (1980) in the context of dynamic error processes. As has often been pointed out, an important advantage of the latter test is that it only requires estimation under the null hypothesis. This is of importance in our case since the autocorrelated system is costly to estimate, as will become apparent in this paper.

More recently, Anderson and Blundell (1982, 1983, 1984) proposed a dynamic allocation system of the form:

$$\Delta y_t = B\,\Delta x_t + \bar{R}(y_{t-1} - \Pi x_{t-1}) + \varepsilon_t$$

which includes, when $B = \Pi$, the autocorrelated system formulated in *levels*. The constraint that $B = \Pi$ implies the equality of short- and long-run coefficients and has been strongly rejected by the data in Anderson and Blundell (1984). Perhaps for this reason, the more general dynamic framework has since been adopted by most authors (e.g. Nakamura, 1986; Veall and Zimmermann, 1986).

However, if a vector autoregressive error structure is appended to a *differential* demand system, such as the Rotterdam system, the resulting model will not be nested within the Anderson–Blundell specification. Moreover, it offers the advantage of parsimony, especially when the dynamic specification involves more than one lag; and the differential form is likely to avoid the serious potential problems caused by unit roots (Granger and Newbold, 1974; Bewley and Elliott, 1992). Such problems will indeed be encountered in the empirical part of this paper. For these reasons, we feel that the empirical evidence on autocorrelation in demand systems needs to be re-examined in the context of differential models.

The aim of this paper is twofold. We will first present a general, and explicit, estimation procedure for the ML estimation of an allocation system with autocorrelated errors, homogeneity, and symmetry. This procedure contains, as a special case, the estimation of the system without the regularity constraints. We will then present a generic procedure, based on the LM principle, for testing any possible combination of absence of serial correlation, homogeneity, and symmetry against any possible alternative which specifies autocorrelation of an arbitrary given order. We view this as important for the following reasons. It is entirely possible that homogeneity and symmetry introduce autocorrelation into an otherwise spherical model; and it was illustrated by the previous authors that misspecified dynamics can severely bias towards rejection the statistics for homogeneity and symmetry. Indeed, if the maintained hypothesis of no autocorrelation is incorrect, it follows from the work of White (1982) that classical tests will generally be of incorrect size; in the present context, a commonsense explanation is also provided by the biased estimated standard errors. On the other hand, if autocorrelation is not present, testing for regularity in the static model is correct and simpler to perform; the tests will presumably be more powerful than their dynamic counterparts; and in the case of homogeneity, a small sample test is available (Laitinen, 1978).

When the regularity restrictions are homogeneity and symmetry, a list of all possible tests of H_0 against H_1 is provided in Table I. In the row and column headings of Table I, A denotes the *absence* of autocorrelation; H denotes homogeneity; S denotes symmetry; and the bars denote logical negation. The list involves 12 tests rather than $8 \times 8 = 64$, since symmetry and adding-up imply homogeneity and since H_0 must be nested within H_1.

Table I. List of possible joint tests

		H_0				
		AHS	$AH\bar{S}$	$A\bar{H}\bar{S}$	$\bar{A}HS$	$\bar{A}H\bar{S}$
H_1	$AH\bar{S}$	Test no. 1				
	$A\bar{H}\bar{S}$	Test no. 2	Test no. 3			
	$\bar{A}HS$	Test no. 4				
	$\bar{A}H\bar{S}$	Test no. 5	Test no. 6		Test no. 7	
	$\bar{A}\bar{H}\bar{S}$	Test no. 8	Test no. 9	Test no. 10	Test no. 11	Test no. 12

The tests in Table I can be divided in three groups. Tests 1–3 are tests for regularity in the static model. Tests 7, 11, and 12 are their dynamic counterparts. The remaining six are (possibly joint) tests for autocorrelation. The LM tests in the first three columns of the table require significantly less computation than the tests in the last two. In fact, it will be shown that the generic formula for the twelve tests reduces to an easily computed trace in cases 1–3, 5, 6, 8, 9, and 10.

Table I raises the obvious question of a strategy for testing the regularity restrictions (HS) of economic theory. In this instance, the empirical findings of previous authors makes it sensible to emphasize protection against an incorrect size due to misspecified dynamics. If the model is indeed regular, Tests 4–6 may be more apt to detect autocorrelation when it is present, since they are based on smaller alternatives than Tests 8–10. Hence an investigator wishing to guard against the common (and presumably improper) rejection of regularity in the static model should not perform tests 1, 2, or 3 unless *none* of the tests 4, 5, 6, 8, 9, and 10 reject. Bonferroni adjustments can be made to control the overall significance level in the latter, six-member group. This discussion admittedly ignores the pre-testing problem inherent in the suggested procedure; but the problem of dealing with 'multiple diagnostics' is known to be difficult even in simple cases (see e.g., Hillier, 1991).

A brief plan of the paper follows. In Section 2 we will present generic expressions for the maximum likelihood estimation of an allocation system under the six possible combinations of constraints (AHS, $AH\bar{S}$, $A\bar{H}\bar{S}$, $\bar{A}HS$, $\bar{A}H\bar{S}$, $\bar{A}\bar{H}\bar{S}$). Generic formulas for the gradient of the loglikelihood and for the information matrix are given in Section 3. The generic LM test statistic is stated, and shown to simplify considerably in cases 1–3, 5, 6, 8, 9, and 10 of Table I (this result was previously shown by Berndt and Savin, 1977, only in the first three cases, where the model is linear under the alternative and the null). A simplification also occurs when A is interpreted as autocorrelation with diagonal, but not necessarily null, matrices of correlation coefficients. In this case the model is only slightly more difficult to estimate than the fully uncorrelated model (the Aitken transformation of the data matrices can be performed elementwise, in Cochrane–Orcutt fashion). Section 4 illustrates the methodology by estimating the AIDS model (Deaton and Muellbauer, 1980a) in levels and in differences, and the Rotterdam model (Theil, 1975; Barten, 1969) with quarterly British data. The most apparent empirical findings are the following: first, the disturbances of the static AIDS model in levels appear to be non-stationary; second, there can be substantial reductions in the statistics for regularity when autocorrelation is allowed for. This observation confirms the empirical findings of the previous authors, albeit in the new context of differential demand systems. Section 5 concludes.

2. RESTRICTED AND UNRESTRICTED MAXIMUM LIKELIHOOD ESTIMATION

As shown by Berndt and Savin (1975), an autocorrelated allocation system satisfying the maintained (and untestable) restriction of adding-up may be written without loss of generality as:

$$Y = BX + U \tag{1}$$

$$U = \sum_{j=1}^{p} R_j U_{-j} + E \tag{2}$$

where Y is an $n \times T$ matrix of T observations on n dependent variables, B is an $n \times k$ matrix of coefficients, X is a $k \times T$ matrix of T observations on k regressors (which could, as in Davidson and MacKinnon, 1980, include lagged dependent variables), U and E are $n \times T$ matrices of current disturbances, the R_j are $n \times n$ matrices of autocorrelation coefficients, and the U_{-j} are $n \times T$ matrices of lagged disturbances. We will assume that $\operatorname{vec} E \sim N(0, I_T \otimes \Sigma)$, where Σ is a positive definite matrix of order n. It is emphasized that equations (1) and (2) are interpreted as an *incomplete* allocation system, after the deletion of an equation; so that, typically, n is the total number of commodities in a demand system minus one.[1] Upon substituting $U_{-j} = Y_{-j} - BX_{-j}$ and equation (1) into equation (2), we obtain:

$$Y = \sum_{j=1}^{p} R_j Y_{-j} + BX - \sum_{j=1}^{p} R_j B X_{-j} + E \tag{3}$$

Upon letting:

$$R = (R_1 \;\; R_2 \;\; \dots \;\; R_p) \tag{4}$$

$$Y_1 = \begin{pmatrix} Y_{-1} \\ Y_{-2} \\ \vdots \\ Y_{-p} \end{pmatrix} \tag{5}$$

$$X_1 = \begin{pmatrix} X_{-1} \\ X_{-2} \\ \vdots \\ X_{-p} \end{pmatrix} \tag{6}$$

$$U = Y - BX \tag{7}$$

$$U_1 = \begin{pmatrix} U_{-1} \\ U_{-2} \\ \vdots \\ U_{-p} \end{pmatrix} = \begin{pmatrix} Y_{-1} - BX_{-1} \\ Y_{-2} - BX_{-2} \\ \vdots \\ Y_{-p} - BX_{-p} \end{pmatrix} = Y_1 - (I_p \otimes B) X_1 \tag{8}$$

we may rewrite equation (3) as:

$$U = RU_1 + E \tag{9}$$

[1] If R_j^* denotes the jth matrix of autocorrelation coefficients in the *full* system, we have:

$$R_j = (I_n \;\; O_{n \times 1}) R_j^* (I_n \;\; -\iota_n)'$$

In the sequel, we denote by I_r an identity matrix of order r, by $O_{r \times s}$ a null matrix with r rows and s columns, and by ι_r a column vector of r ones. 'vec' denotes the stack operator and 'tr' denotes trace. The identities $\operatorname{vec}(ABC) = (C' \otimes A) \operatorname{vec} B$ and $\operatorname{tr}(AB) = (\operatorname{vec} A')' \operatorname{vec} B$ will be used repeatedly.

Similarly, if we note that:

$$\text{vec}\left(\sum_{j=1}^{p} R_j B X_{-j}\right) = \sum_{j=1}^{p} (X'_{-j} \otimes R_j)\text{vec } B$$

and that $\text{vec}(BX) = (X' \otimes I_n)\text{vec } B$, we may express equation (3) as:

$$\text{vec}(Y - RY_1) = \left[(X' \otimes I_n) - \sum_{j=1}^{p} (X'_{-j} \otimes R_j)\right]\text{vec } B + \text{vec } E \tag{10}$$

Since vec E is multivariate normal, the loglikelihood corresponding to equation (3) can be written as:

$$L(B, R, \Sigma) = -\frac{nT}{2}\log 2\pi - \frac{T}{2}\log\det\Sigma$$
$$-\tfrac{1}{2}\text{tr } \Sigma^{-1}[Y - RY_1 - BX + R(I_p \otimes B)X_1][Y - RY_1 - BX + R(I_p \otimes B)X_1]'$$

which is a special case of Hendry (1971, eq. 6). Equation (9) implies that for given B, the model is a reduced form with coefficient matrix R; and equation (10) implies that for given R, it is a multivariate regression with coefficient vector vec B. From standard results on maximum likelihood estimation, it follows that the ML estimator of R is:

$$\hat{R} = \hat{U}\hat{U}_1'(\hat{U}_1\hat{U}_1')^{-1} \tag{11}$$

with $\hat{U} = Y - \hat{B}X$ and $\hat{U}_1 = Y_1 - (I_p \otimes \hat{B})X_1$, $\hat{B}$ being the ML estimator of B. In order to impose homogeneity and symmetry on vec B in equation (10) we use the methodology presented in Deschamps (1988). If we let $B = (C\ \ S\ \ s)$, where $(S\ \ s)$ is an $n \times (n+1)$ matrix of price coefficients and where s is the last column of B, homogeneity and symmetry are stated, respectively, as $S\iota_n = -s$ and $S = S'$. We define:

$$D_H = \begin{pmatrix} I_{k-1} & \begin{matrix} O_{(k-n-1)\times 1} \\ -\iota_n \end{matrix} \end{pmatrix} \tag{12}$$

$$D_S = \begin{pmatrix} I_{n(k-n-1)} & O \\ O & L \end{pmatrix} \tag{13}$$

where L is an $n^2 \times n(n+1)/2$ matrix such that vec S equals L times the stacked lower triangle of S (for an explicit form of L, see Balestra, 1976; and for an algorithm generating a compact computer representation of L, see Deschamps, 1988). In the case where S is 2×2, L has the following form:

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

With these definitions, we may impose the homogeneity and symmetry restrictions on vec B as:

$$\text{vec } B = (D_H' \otimes I_n)D_S b \tag{14}$$

where

$$b' = (\text{vec}'C\ \ S_{11}\ \ S_{21}\ \ S_{22}\ \ S_{31}\ \ S_{32}\ \ S_{33}\ \ \ldots\ \ S_{nn})$$

is a vector of unconstrained parameters.

Upon substituting equation (14) into the regression equation (10) we see that the ML estimator of b is given by generalized least squares as:

$$\hat{b} = [\hat{Z}'(I_T \otimes \hat{\Sigma}^{-1})\hat{Z}]^{-1}\hat{Z}'(I_T \otimes \hat{\Sigma}^{-1})\text{vec}(Y - \hat{R}Y_1) \tag{15}$$

with

$$\hat{Z} = \left[(X' \otimes I_n) - \sum_{j=1}^{p} (X'_{-j} \otimes \hat{R}_j)\right](D'_H \otimes I_n)D_S$$

$$= \left[(X'D'_H \otimes I_n) - \sum_{j=1}^{p} (X'_{-j}D'_H \otimes \hat{R}_j)\right]D_S \tag{16}$$

$$\hat{\Sigma} = \hat{E}\hat{E}'/T \tag{17}$$

$$\hat{E} = Y - \hat{R}Y_1 - \hat{B}X + \hat{R}(I_p \otimes \hat{B})X_1 \tag{18}$$

where $\hat{R}$ is given by equation (11), where $\hat{R}_j$ is the appropriate block of $\hat{R}$, and where vec $\hat{B} = (D'_H \otimes I_n)D_S\hat{b}$.

It is straightforward to check that equation (15) can be simplified as:

$$\hat{b} = [\hat{Z}'(I_T \otimes \hat{\Sigma}^{-1})\hat{Z}]^{-1}D'_S \text{ vec}\left[\hat{\Sigma}^{-1}(Y - \hat{R}Y_1)X'D'_H - \sum_{j=1}^{p} \hat{R}'_j\hat{\Sigma}^{-1}(Y - \hat{R}Y_1)X'_{-j}D'_H\right] \tag{19}$$

In order to impose homogeneity only, it suffices to replace D_S in equation (14) by an identity matrix of order $n(k-1)$, which reduces equation (14) to $B = B_H D_H$ with $B_H = (C\ S)$. Of course, b is then redefined as vec B_H. Similarly, equation (14) implies unconstrained estimation when D_S and D_H are replaced by identity matrices; b is in this case redefined as vec B. We summarize our results in the following theorem.

Theorem 1. *The maximum likelihood estimation of equations (1) and (2) under homogeneity and symmetry requires the solution of equations (11) and (16)–(19) with vec $\hat{B} = (D'_H \otimes I_n)D_S\hat{b}$, and D_H, D_S given by equations (12) and (13). Estimation under homogeneity requires the solution of equations (11) and (16)–(19) with vec $\hat{B} = (D'_H \otimes I_n)\hat{b}$, with D_H given by equation (12) and with $D_S = I_{n(k-1)}$. Unconstrained estimation requires the solution of equations (11) and (16)–(19) with vec $\hat{B} = \hat{b}$, $D_H = I_k$ and $D_S = I_{nk}$.*

If we assume uncorrelated disturbances, equation (11) is replaced by $\hat{R} = O$, and formulas (18) and (19) are considerably simplified (see Deschamps, 1988). The system also becomes much simpler if we assume that $R_j = \rho_j I_n$, since equation (9) may be written in this case as:

$$U = \sum_{j=1}^{p} \rho_j U_{-j} + E$$

with U and U_{-j} given by equations (7) and (8). The ML estimates of the ρ_j are then the GLS coefficient estimates in the regression of vec $\hat{U}$ on (vec $\hat{U}_{-1}, \ldots,$ vec $\hat{U}_{-p}$). Furthermore, it is straightforward to check that equations (18) and (19) respectively simplify to:

$$\hat{E} = Y_* - \hat{B}X_* \tag{18a}$$

$$\hat{b} = [D'_S(D_H X_* X'_* D'_H \otimes \hat{\Sigma}^{-1})D_S]^{-1}D'_S \text{ vec}(\hat{\Sigma}^{-1}Y_* X'_* D'_H) \tag{19a}$$

with $Y_* = Y - \Sigma_{j=1}^{p}\hat{\rho}_j Y_{-j}$ and $X_* = X - \Sigma_{j=1}^{p}\hat{\rho}_j X_{-j}$.

3. JOINT TESTS FOR AUTOCORRELATION, HOMOGENEITY, AND SYMMETRY

It is well known that for linear models of the form $z = Zb + \text{vec } E$, with $\text{vec } E \sim N(0, I_T \otimes \Sigma)$, the gradient of the (unconcentrated) loglikelihood $L(b, \Sigma)$ with respect to b is given by:

$$\frac{\partial L}{\partial b} = Z'(I_T \otimes \Sigma^{-1})(z - Zb) = Z' \text{ vec}(\Sigma^{-1}E)$$

The information matrix is also well known to be block-diagonal. Under regularity assumptions, the block of this matrix corresponding to b is:

$$\mathcal{I}_{bb} = \mathcal{E}\left(\frac{\partial L}{\partial b}\frac{\partial L}{\partial b'}\right) = Z'(I_T \otimes \Sigma^{-1})Z$$

When the regression equation is non-linear, as in equation (3), it follows that if we may write:

$$z = Zb + \text{vec } E \qquad \text{and} \qquad z_1 = Z_1 c + \text{vec } E$$

where (z, Z) does not involve b and (z_1, Z_1) does not involve c, then the gradient of the loglikelihood with respect to $\alpha = (b, c)$ is equal to:

$$\frac{\partial L}{\partial \alpha} = \begin{pmatrix} Z' \text{ vec}(\Sigma^{-1}E) \\ Z_1' \text{ vec}(\Sigma^{-1}E) \end{pmatrix}$$

and that the first diagonal block of the information matrix is:

$$\begin{aligned} \mathcal{I}_{\alpha\alpha} &= \begin{bmatrix} Z'(I_T \otimes \Sigma^{-1})Z & Z'(I_T \otimes \Sigma^{-1})Z_1 \\ Z_1'(I_T \otimes \Sigma^{-1})Z & Z_1'(I_T \otimes \Sigma^{-1})Z_1 \end{bmatrix} \\ &= \begin{pmatrix} \mathcal{I}_{bb} & \mathcal{I}_{bc} \\ \mathcal{I}_{bc}' & \mathcal{I}_{cc} \end{pmatrix} \end{aligned} \tag{20}$$

It is clear, upon examination of equations (9), (10), and (14), that the preceding developments apply to our model if we define b as in Section 2, $c = \text{vec } R$, Z as in equation (16), and $Z_1 = (U_1' \otimes I_n)$. It follows that:[2]

$$\frac{\partial L}{\partial \alpha} = \begin{bmatrix} D_S' \text{ vec}(\Sigma^{-1}EX'D_H' - \Sigma_{j=1}^{p} R_j'\Sigma^{-1}EX_{-j}'D_H') \\ \text{vec } \Sigma^{-1} \ EU_1' \end{bmatrix} = \begin{pmatrix} \partial L/\partial b \\ \partial L/\partial c \end{pmatrix} \tag{21}$$

$$\mathcal{I}_{bb} = D_S'\left[(D_H X \otimes \Sigma^{-1}) - \sum_{j=1}^{p} (D_H X_{-j} \otimes R_j'\Sigma^{-1})\right] \left[(X'D_H' \otimes I_n) - \sum_{j=1}^{p} (X_{-j}'D_H' \otimes R_j)\right] D_S \tag{22}$$

$$\mathcal{I}_{bc} = D_S'\left[(D_H X U_1' \otimes \Sigma^{-1}) - \sum_{j=1}^{p} (D_H X_{-j} U_1' \otimes R_j'\Sigma^{-1})\right] \tag{23}$$

$$\mathcal{I}_{cc} = U_1 U_1' \otimes \Sigma^{-1} \tag{24}$$

Our generic Lagrange multiplier test statistic follows immediately from the above results.

---

[2] Following Breusch and Pagan (1980), we interpret the expectations in $\mathcal{I}_{bc}$ and $\mathcal{I}_{cc}$ as being conditional on U_1.

Following Breusch and Pagan (1980), it may be written as:

$$LM = \left(\frac{\partial L}{\partial \alpha}(\hat{\theta}_0)\right)' \mathscr{I}_{\alpha\alpha}^{-1}(\hat{\theta}_0)\left(\frac{\partial L}{\partial \alpha}(\hat{\theta}_0)\right) \qquad \text{if } H_1 \text{ includes } \bar{A}$$
$$= \left(\frac{\partial L}{\partial b}(\hat{\theta}_0)\right)' \mathscr{I}_{\bar{b}b'}^{-1}(\hat{\theta}_0)\left(\frac{\partial L}{\partial b}(\hat{\theta}_0)\right) \qquad \text{if } H_1 \text{ includes } A \tag{25}$$

where $\hat{\theta}_0$ denotes the ML estimate of $(U_1 \;\; R \;\; E \;\; \Sigma)$ under the null hypothesis. For the 12 tests mentioned in Table I of Section 1 the dimension of b in equation (25) will vary according to the alternative H_1, with $b = \text{vec}\, B$, $b = \text{vec}(C \;\; S)$, or $b' = (\text{vec}'C \;\; S_{11} \;\; S_{12} \;\; S_{22} \; \ldots \; S_{nn})$ when H_1 specifies $\bar{H}\bar{S}$, $H\bar{S}$, or HS, respectively. Since, as was seen in Section 2, the specification of $\bar{H}\bar{S}$ and $H\bar{S}$ result from replacing D_S and D_H by I_{nk} and I_k in the first case and D_S by $I_{n(k-1)}$ in the second case, we have the following theorem.

Theorem 2. *The LM test statistic for the twelve cases in Table I is given by equation (25). $(\partial L/\partial\alpha)(\hat{\theta}_0)$, $\mathscr{I}_{\alpha\alpha}(\hat{\theta}_0)$, $(\partial L/\partial b)(\hat{\theta}_0)$, and $\mathscr{I}_{bb}(\hat{\theta}_0)$ are generated according to the following rules:*

(1) Let $\hat{B}_0$ be the ML estimate of B under H_0, and let $\hat{U} = Y - \hat{B}_0 X$. The matrices U_1, R, E, and Σ in equations (20)–(24) are estimated by:

$$\hat{U}_1 = Y_1 - (I_p \otimes \hat{B}_0)X_1$$
$$\hat{R} = \hat{U}\hat{U}_1'(\hat{U}_1\hat{U}_1')^{-1} \qquad \text{if } H_0 \text{ includes } \bar{A}$$
$$= O \qquad \text{if } H_0 \text{ includes } A$$
$$\hat{E} = \hat{U} - \hat{R}\hat{U}_1$$
$$\hat{\Sigma} = \hat{E}\hat{E}'/T$$

(2) If H_1 includes H, define D_H by equation (12) in equations (21)–(23). Otherwise define $D_H = I_k$ in equations (21)–(23).

(3) If H_1 includes S, define D_S by equation (13) in equations (21)–(23). Otherwise let, in equations (21)–(23), $D_S = I_{nk}$ if H_1 includes $\bar{H}$ and $D_S = I_{n(k-1)}$ if H_1 includes H.

Since in the first three cases of Table I, the model is linear both under the null and under the alternative, we have the following theorem, due to Berndt and Savin (1977).

Theorem 3. *If both H_0 and H_1 include A, then:*

$$LM = T\,tr[\hat{\Sigma}_0^{-1}(\hat{\Sigma}_0 - \hat{\Sigma}_1)] \tag{26}$$

where $\hat{\Sigma}_0$ and $\hat{\Sigma}_1$ are the ML estimators of Σ under H_0 and H_1, respectively.

Equation (26) obviously presents a definite advantage over equation (25) since it only requires the inversion of an $n \times n$ matrix. We will now show that similar simplifications occur in cases 5, 6, 8, 9, and 10 of Table I (the largest matrix to be inverted is of order $\max(k, np)$).

Theorem 4. *Define:*

$$Q_X = X'(XX')^{-1}X$$
$$Q_X^H = X'D_H'(D_H XX'D_H')^{-1}D_H X$$
$$\hat{U} = Y - \hat{B}_0 X$$
$$\hat{U}_1 = Y_1 - (I_p \otimes \hat{B}_0)X_1$$

$$\hat{\Sigma} = \hat{U}\hat{U}'/T$$
$$Q = \hat{U}'\hat{\Sigma}^{-1}\hat{U}$$
$$Q_1 = \hat{U}_1'[\hat{U}_1(I_T - Q_X)\hat{U}_1']^{-1}\hat{U}_1$$
$$Q_1^H = \hat{U}_1'[\hat{U}_1(I_T - Q_X^H)\hat{U}_1']^{-1}\hat{U}_1$$

where $\hat{B}_0$ is the ML estimate of B under H_0. The Lagrange multiplier test statistic (25) is given, for the tests numbered 8–10 in Table I, by:

$$LM = tr[Q(Q_X(I_T + Q_1Q_X - 2Q_1) + Q_1)] \tag{27}$$

and for the tests numbered 5 and 6 in Table I by:

$$LM = tr[Q(Q_X^H(I_T + Q_1^HQ_X^H - 2Q_1^H) + Q_1^H)] \tag{28}$$

Furthermore, for the tests numbered 6 and 10, the statistic simplifies, respectively, to $LM = tr(QQ_1^H)$ and to $LM = tr(QQ_1)$.

Proof: Since equation (28) results from replacing X by D_HX in Q_X and Q_1, it suffices to prove that the LM statistic (25) reduces to equation (27) for the tests numbered 8–10. In these cases H_1 specifies unconstrained estimation, so that Rules 2 and 3 in Theorem 2 prescribe replacing D_S by I_{nk} and D_H by I_k in equations (21)–(23). Furthermore H_0 specifies no serial correlation, so that Rule 1 prescribes replacing R by O in equations (21)–(23). We may then write equations (20) and (21) as:

$$\frac{\partial L}{\partial \alpha} = \begin{pmatrix} \text{vec } \Sigma^{-1}EX' \\ \text{vec } \Sigma^{-1}EU_1' \end{pmatrix} \tag{29}$$

$$\mathcal{I}_{\alpha\alpha} = \begin{pmatrix} XX' & XU_1' \\ U_1X' & U_1U_1' \end{pmatrix} \otimes \Sigma^{-1} \tag{30}$$

By the partitioned inversion formula, we have:

$$\mathcal{I}_{\alpha\alpha}^{-1} = \begin{pmatrix} \mathcal{I}^{bb} & \mathcal{I}^{bc} \\ \mathcal{I}^{cb} & \mathcal{I}^{cc} \end{pmatrix} \otimes \Sigma \tag{31}$$

with

$$\mathcal{I}^{cc} = [U_1(I_T - X'(XX')^{-1}X)U_1']^{-1} \tag{32}$$
$$\mathcal{I}^{bb} = (XX')^{-1}(I_k + XU_1'\mathcal{I}^{cc}U_1X'(XX')^{-1}) \tag{33}$$
$$\mathcal{I}^{bc} = -(XX')^{-1}XU_1'\mathcal{I}^{cc} \tag{34}$$
$$\mathcal{I}^{cb} = (\mathcal{I}^{bc})' \tag{35}$$

It follows that:

$$\begin{aligned}
&\left(\frac{\partial L}{\partial \alpha}\right)' \mathcal{I}_{\alpha\alpha}^{-1} \left(\frac{\partial L}{\partial \alpha}\right) \\
&= (\text{vec}'\Sigma^{-1}EX' \;\; \text{vec}'\Sigma^{-1}EU_1') \begin{pmatrix} \text{vec}(EX'\mathcal{I}^{bb} + EU_1'\mathcal{I}^{cb}) \\ \text{vec}(EX'\mathcal{I}^{bc} + EU_1'\mathcal{I}^{cc}) \end{pmatrix} \\
&= \text{tr}(XE'\Sigma^{-1}EX'\mathcal{I}^{bb} + XE'\Sigma^{-1}EU_1'\mathcal{I}^{cb} + U_1E'\Sigma^{-1}EX'\mathcal{I}^{bc} + U_1E'\Sigma^{-1}EU_1'\mathcal{I}^{cc}) \\
&= \text{tr}(XE'\Sigma^{-1}EX'\mathcal{I}^{bb} + 2XE'\Sigma^{-1}EU_1'\mathcal{I}^{cb} + U_1E'\Sigma^{-1}EU_1'\mathcal{I}^{cc}) \\
&= \text{tr}(E'\Sigma^{-1}EX'\mathcal{I}^{bb}X + 2E'\Sigma^{-1}EU_1'\mathcal{I}^{cb}X + E'\Sigma^{-1}EU_1'\mathcal{I}^{cc}U_1) \\
&= \text{tr}(E'\Sigma^{-1}E(X'\mathcal{I}^{bb}X + 2U_1'\mathcal{I}^{cb}X + U_1'\mathcal{I}^{cc}U_1))
\end{aligned} \tag{36}$$

We now use the definitions in the statement of the theorem, replace U_1 by $\hat{U}_1$, and replace, according to Rule 1 in Theorem 2, E by $\hat{E} = \hat{U} = Y - \hat{B}_0 X$ in equation (36). We see that:

$$X'\mathscr{I}^{bb}X = Q_X + Q_X Q_1 Q_X$$
$$\hat{U}_1'\mathscr{I}^{cb}X = -Q_1 Q_X$$
$$\hat{U}_1'\mathscr{I}^{cc}\hat{U}_1 = Q_1$$

and equation (27) follows from equation (36) and from the symmetry of Q, Q_1, and Q_X, which implies:

$$\operatorname{tr}(QQ_1Q_X) = \operatorname{tr}(Q_1Q_XQ) = \operatorname{tr}(QQ_XQ_1)$$

Lastly, equation (27) simplifies to $\operatorname{tr}(QQ_1)$ for Test 10 in Table I, since in this case $\hat{B}_0 = YX'(XX')^{-1}$ and $\hat{U} = Y(I_T - Q_X)$, so that $QQ_X = O$ in equation (27). This concludes the proof of Theorem 4. □

We conclude this section with two remarks. First, in the case where the system consists of a single equation, it is straightforward to verify that $\operatorname{tr}(QQ_1)$ reduces to TR^2, where R^2 is the coefficient of determination in a regression of the vector of OLS residuals $\hat{u}$ on $(X, \hat{u}_{-1}, \ldots, \hat{u}_{-p})$ (see Breusch and Pagan, 1980). Second, it is easy to redefine the *null* hypothesis A as meaning $R_j = \rho_j I_n$ for all j, rather than $R_j = O$ for all j. It should be pointed out, however, that this redefined null hypothesis is only slightly less restrictive than the total absence of serial correlation. All the results of Theorem 4 can also be seen to apply to this case upon replacing X by $X_* = X - \Sigma_{j=1}^{p}\hat{\rho}_j X_{-j}$ and Y by $Y_* = Y - \Sigma_{j=1}^{p}\hat{\rho}_j Y_{-j}$, as in equations (18a) and (19a). Note, in particular, that $\hat{U} = Y - \hat{B}_0 X$ must be replaced by $\hat{E} = Y_* - \hat{B}_0 X_*$; the two are no longer equal under the redefined null hypothesis.

4. AN EMPIRICAL ILLUSTRATION

In this section we will illustrate the preceding theory with an estimation on quarterly British data of two well-known consumer demand models. The first is the AIDS model (Deaton and Muellbauer, 1980a):

$$w_{it} = \alpha_i + \delta_i t + \sum_j \gamma_{ij} \log p_{jt} + \beta_i \log(x_t/P_t) + u_{it} \tag{37}$$

where p_{it} denotes the price of commodity i at time t; $x_t = \Sigma_j p_{jt} q_{jt}$ is total expenditure; $w_{it} = p_{it} q_{it}/x_t$ is the budget share of commodity i; and $\log P_t = \Sigma_j \bar{w}_j \log p_{jt}$, with $\bar{w}_j = T^{-1}\Sigma_{t=1}^{T} w_{jt}$.[3] The second model is the Rotterdam system (Theil, 1975; Barten, 1969), given by:

$$\bar{w}_{it}\, \Delta \log q_{it} = a_i + b_i \sum_j \bar{w}_{jt}\, \Delta \log q_{jt} + \sum_j c_{ij}\, \Delta \log p_{jt} + u_{it}^* \tag{38}$$

where $\bar{w}_{it} = (w_{it} + w_{i,t-4})/2$, $\Delta \log q_{it} = \log q_{it} - \log q_{i,t-4}$, and $\Delta \log p_{it} = \log p_{it} - \log p_{i,t-4}$. A differential form of the AIDS model is:

$$\Delta w_{it} = \delta_i + \sum_j \gamma_{ij}\, \Delta \log p_{jt} + \beta_i\, \Delta \log(x_t/P_t) + \varepsilon_{it} \tag{39}$$

where Δ again denotes the seasonal difference operator.

[3] The sample averages of the budget shares are used in order to eliminate potential simultaneity problems.

The quarterly expenditures on nine classes of consumption goods (Food; Alcoholic Drink and Tobacco; Housing; Fuel and Light; Clothing and Footwear; Durable Household Goods; Cars and Motorcycles; Other Goods; Other Services) are available in the *Economic Trends Annual Supplement*, 1983, for the period from 1955 (quarter 1) to 1982 (quarter 2). The nine corresponding average budget shares are 0·21, 0·12, 0·12, 0·05, 0·09, 0·05, 0·03, 0·15 and 0·18. Corresponding price indices are available in the *Monthly Digest of Statistics*. (In the case of 'Cars and Motorcycles', this is the 'Transport and Vehicles' price index.)

When durable goods are included in demand systems, the neoclassical tradition calls for including in the utility function the *stocks* of the durable commodities, from the decision period to the end of the planning horizon; and to define the corresponding prices of durable goods as rental equivalent prices, or user costs. This approach is followed by Muellbauer (1981), who estimates demand equations for durables and non-durables using a dynamic extension of the linear expenditure system. Muellbauer reports a strong rejection of the restrictions implied by the neoclassical model, and suggests as possible reasons the presence of uncertainty and transactions costs (which are not accounted for in the neoclassical analysis). Another possible reason, mentioned by Deaton and Muellbauer (1980b, p. 207), is the element of arbitrariness introduced by generating series on stocks of durable goods from expenditure data and assumed depreciation rates.

A more common approach is to estimate equations (37)–(39) for non-durable goods only, and to justify the omission of durables by assuming the separability of the utility function. However, this assumption is implausible, and reflects only the investigator's inability to model adequately the dynamics in the durables equations.

For these reasons, we venture to say that in the absence of an empirically practical generalization of the neoclassical model for durables that avoids the shortcomings highlighted in Muellbauer (1981), versions of equations (37)–(39) that treat all commodities (durable and non-durable) symmetrically are defensible, provided that one corrects any resulting dynamic misspecification by appropriately modelling the disturbances as stochastic processes. We therefore estimated equations (37)–(39) using expenditure and purchase price data on all nine commodities, including durable ones.

Table II reports the income elasticities (ε_i) and compensated own-price elasticities (η_{ii}) estimated under homogeneity from the following specifications:[4]

Model 1: equation (37), vec $U \sim N(0, I_T \otimes \Sigma)$;
Model 2: equation (39), vec $U \sim N(0, I_T \otimes \Sigma)$;
Model 3: equation (38), vec $U \sim N(0, I_T \otimes \Sigma)$;
Model 2A: equation (39), $U = R_1U_{-1} + R_4U_{-4} + E$, vec $E \sim N(0, I_T \otimes \Sigma)$;
Model 3A: equation (38), $U = R_1U_{-1} + R_4U_{-4} + E$, vec $E \sim N(0, I_T \otimes \Sigma)$.

Therefore Model 1 is the spherical AIDS model in levels, Models 2 and 2A are the spherical and autocorrelated AIDS models in differences, Models 3 and 3A are the spherical and autocorrelated Rotterdam models. In Models 2A and 3A, the intermediate lags were omitted in order to keep the number of estimated parameters within reasonable bounds, and because the other possible two-lag structures would not be economically meaningful. The omission of intermediate lags involves only a trivial redefinition of the matrices R, X_1, Y_1, and U_1 in Sections 2 and 3 (for instance, R becomes $(R_1 \; R_4)$).

[4] An approximate correspondence between the income and own-price coefficients in equations (37) and (38) is given by $b_i = \beta_i + \bar{w}_i$ and by $c_{ii} = \gamma_{ii} - \bar{w}_i + \bar{w}_i^2$. Elasticities are obtained upon dividing by the average budget shares $\bar{w}_i$.

Table II. Elasticity estimates under homogeneity

	Model 1		Model 2		Model 3		Model 2A		Model 3A	
	ε_i	η_{ii}	ε_i	η_{ii}	ε_i	η_{ii}	ε_i	η_{ii}	ε_i	η_{ii}
Food	0·45	−0·61	0·49	−0·47	0·41	−0·46	0·53	−0·56	0·54	−0·58
	11·83	−12·81	7·47	−9·48	5·98	−8·95	7·22	−10·98	6·98	−11·03
Drink and Tobacco	2·37	−0·18	0·59	−0·59	0·56	−0·57	0·77	−0·54	0·75	−0·45
	21·47	−1·41	5·50	−9·03	5·24	−8·81	7·00	−7·84	6·76	−6·75
Housing	0·28	−0·49	0·41	−0·55	0·50	−0·67	0·37	−0·60	0·53	−0·64
	5·12	−7·54	3·54	−7·92	4·16	−9·41	3·51	−7·22	4·74	−7·97
Fuel and Light	−1·64	2·81	0·19	0·03	0·23	0·09	0·23	−0·15	0·24	−0·12
	−3·33	4·00	0·73	0·14	0·87	0·52	0·81	−0·81	0·83	−0·66
Clothing	3·43	−1·08	1·21	−0·36	1·12	−0·24	1·26	−0·42	1·11	−0·24
	21·20	−4·44	10·55	−2·95	9·76	−2·01	9·63	−3·29	8·46	−1·86
Durables	2·35	−0·27	3·06	−1·61	3·14	−1·54	3·15	−1·80	3·09	−1·55
	14·10	−0·55	13·26	−5·30	13·71	−5·18	13·60	−5·88	13·24	−5·30
Vehicles	−2·27	−1·79	8·57	−1·58	9·16	−1·67	9·48	−2·58	9·51	−3·02
	−3·76	−1·38	12·25	−2·22	13·36	−2·40	15·97	−4·29	15·83	−5·37
Other Goods	1·52	−1·18	0·60	−0·91	0·61	−0·92	0·72	−0·61	0·74	−0·61
	21·71	−6·65	7·07	−9·32	6·86	−9·02	12·49	−8·09	12·20	−7·53
Other Services	0·59	−0·75	0·78	−0·47	0·73	−0·46	0·32	−0·31	0·28	−0·29
	3·63	−2·38	8·89	−6·30	8·05	−6·10	4·05	−4·08	3·66	−3·75

Here and in Table III model 1 is the uncorrelated AIDS model in levels; Models 2 and 2A are the uncorrelated and autocorrelated AIDS models in differences; Models 3 and 3A are the uncorrelated and autocorrelated Rotterdam models. The figures show the estimated income elasticities (ε_i) and the estimated own-price compensated elasticities (η_{ii}) followed by the ratios to their estimated asymptotic standard errors.

It is immediately apparent from the first two columns of Table II that the spherical AIDS model in levels yields implausible results. 'Fuel and Light' and 'Vehicles' are classified as inferior goods. 'Durables' are price inelastic, and the price elasticity for 'Fuel and Light' has the wrong sign. The LM test statistic for autocorrelation when homogeneity is maintained (Test 6) is equal to 415·57; this compares with a critical value of 168·13 for the chi-square distribution with 128 degrees of freedom. The residuals from Model 1 exhibit quite severe autocorrelation, with a seasonal Durbin–Watson statistic as low as 0·39 for the 'Fuel and Light' equation. The regularity test statistics are very large: Test 2 yields LR = 151·44 and LM = 127·81, whereas the critical value of the chi-square distribution with 36 degrees of freedom is 58·62 at the 1% significance level. An unsuccessful attempt to estimate equation (37) under homogeneity with autocorrelated errors indicates that the problem may be caused by non-stationary disturbances. None of the optimization algorithms that were tried converged; the information matrix became singular, and the likelihood function discontinuous, during the iterations. This problem can also occur in the single-equation model if the autocorrelation coefficient becomes arbitrarily close to one, and has also been encountered in other contexts; see Quandt (1983, p. 744). In the single-equation model a penalty function could be introduced in the likelihood to prevent this occurrence. It is difficult, however, to generalize this technique to the multivariate regression case, where the stationarity conditions involve the several (possibly complex) eigenvalues of the autocorrelation matrices (see Berndt and Savin, 1975).

The next four columns in Table II tell a different story. The estimated elasticities for the Rotterdam and differential AIDS systems are quite close. The same is true when allowance is made for autocorrelation (last four columns in Table II). However, there can be substantial differences between the spherical and autocorrelated versions: taking autocorrelation into account more than halves the income elasticity estimate for 'Other Services', and almost doubles the price elasticity estimate for 'Vehicles'. The t-ratios can also be markedly different in the spherical and autocorrelated versions, especially for the last three commodities. Similar observations can be made (more emphatically) in Table III, where the models are estimated under symmetry. In this case the spherical models 2 and 3 classify 'Vehicles' as price inelastic, an implausible result since the elasticity estimates are short run rather than long run (the data on durable goods refer to flows rather than stocks).

We now turn to a discussion of the test statistics for autocorrelation and/or regularity. Table IV presents the 12 LM and LR statistics for the differential AIDS and Rotterdam models (2, 2A, 3, and 3A). We first note that the test statistics for homogeneity are considerably lower in the autocorrelated models; this result is consistent with, for instance, Anderson and Blundell (1983, 1984). Test 3 rejects homogeneity in all cases at the 5% level, whereas the statistics for Test 12 are insignificant at the 38% level. A small sample correction for Test 3 does not significantly change this result. When the correction of Anderson (1958, p. 208), which has a rigorous theoretical basis, is applied to the LR statistics of Test 3, $LR = 18{\cdot}90$ is deflated to $16{\cdot}23$, and $LR = 21{\cdot}23$ is deflated to $18{\cdot}22$, both remaining significant at the 5% level. The exact test of Laitinen (1978) gives the same results, with F-statistics of 2·147 for the AIDS model and 2·439 for the Rotterdam model. With 8 and 88 degrees of freedom, both values are significant at the 5% level.

A similar observation holds for the LM tests of symmetry. In the joint regularity test, the reduction is sufficient to pull the two LM statistics out of the 1% critical region (compare Test 2 and Test 11). However, the two LM statistics in Test 7 are significant at the 1% level.

The reductions in the LR statistics for symmetry are less significant, and LR actually

Table III. Elasticity estimates under symmetry

	Model 1		Model 2		Model 3		Model 2A		Model 3A	
	ε_i	η_{ii}	ε_i	η_{ii}	ε_i	η_{ii}	ε_i	η_{ii}	ε_i	η_{ii}
Food	0·44	−0·48	0·51	−0·46	0·48	−0·44	0·65	−0·48	0·68	−0·47
	11·95	−17·99	8·30	−9·90	7·43	−9·18	8·98	−11·46	9·15	−9·76
Drink and Tobacco	2·24	−0·46	0·66	−0·60	0·64	−0·57	0·71	−0·64	0·65	−0·55
	20·22	−5·77	6·63	−9·04	6·45	−8·66	6·53	−9·33	5·90	−8·43
Housing	0·27	−0·57	0·48	−0·57	0·60	−0·67	0·37	−0·52	0·65	−0·49
	4·72	−11·84	4·51	−8·81	5·51	−10·13	3·21	−6·51	5·61	−6·26
Fuel and Light	−1·45	−0·12	0·12	−0·12	0·17	−0·05	−0·04	−0·17	0·08	−0·14
	−2·87	−0·23	0·47	−0·69	0·71	−0·27	−0·14	−0·98	0·28	−0·85
Clothing	3·30	−0·90	1·33	−0·41	1·23	−0·28	1·40	−0·41	1·29	−0·32
	20·17	−6·41	11·45	−3·81	10·53	−2·66	10·42	−3·43	9·79	−2·67
Durables	2·42	−1·54	2·96	−2·04	2·98	−2·06	3·22	−2·29	2·98	−2·51
	13·91	−5·39	13·16	−7·88	13·32	−8·12	12·83	−8·56	11·91	−10·01
Vehicles	−1·70	−2·04	6·93	−0·68	7·20	−0·79	8·64	−2·55	7·95	−2·41
	−2·67	−2·48	10·56	−0·98	11·02	−1·12	13·45	−4·19	12·40	−3·99
Other Goods	1·47	−0·91	0·72	−0·74	0·72	−0·76	0·69	−0·65	0·74	−0·69
	20·49	−9·21	8·67	−8·55	8·56	−8·69	11·21	−8·53	11·79	−8·95
Other Services	0·61	−0·47	0·85	−0·50	0·81	−0·46	0·39	−0·39	0·37	−0·33
	3·61	−2·14	10·59	−7·27	9·85	−6·48	4·73	−5·04	4·75	−4·37

Table IV. Test statistics

Test no.	H_0	H_1	AIDS (Diff.) LR	AIDS (Diff.) LM	Rotterdam LR	Rotterdam LM	CV1	CV5
1	AHS	$AH\bar{S}$	81·07	72·91	86·50	76·90	48·28	41·34
2	AHS	$A\bar{H}\bar{S}$	99·97	89·27	107·73	95·50	58·62	51·00
3	$AH\bar{S}$	$A\bar{H}\bar{S}$	18·90	17·31	21·23	19·24	20·09	15·51
4	AHS	$\bar{A}HS$	472·23	321·28	454·69	313·68	168·13	155·40
5	AHS	$\bar{A}H\bar{S}$	552·52	346·84	545·28	345·79	200·01	186·15
6	$AH\bar{S}$	$\bar{A}H\bar{S}$	474·51	300·16	462·04	292·13	168·13	155·40
7	$\bar{A}HS$	$\bar{A}H\bar{S}$	80·29	50·54	90·59	54·86	48·28	41·34
8	AHS	$\bar{A}\bar{H}\bar{S}$	558·05	352·51	551·55	349·99	209·05	194·88
9	$AH\bar{S}$	$\bar{A}\bar{H}\bar{S}$	480·04	305·93	468·31	297·04	177·28	164·22
10	$A\bar{H}\bar{S}$	$\bar{A}\bar{H}\bar{S}$	461·85	293·91	447·89	281·57	168·13	155·40
11	$\bar{A}HS$	$\bar{A}\bar{H}\bar{S}$	85·82	54·20	96·86	58·24	58·62	51·00
12	$\bar{A}H\bar{S}$	$\bar{A}\bar{H}\bar{S}$	5·53	4·11	6·27	5·01	20·09	15·51

A denotes no autocorrelation, H denotes homogeneity, S denotes symmetry, bars denote logical negation. CV1 and CV5 are the critical values at the 1% and 5% significance levels.

As implied by the theory in Berndt and Savin (1977), Savin (1976), and Breusch (1979), the LM statistics for regularity in Tests 1–3, 7, 11, and 12 are all lower than their LR counterparts.

We also note that all the regularity statistics (1–3, 7, 11, and 12) are lower for AIDS than for Rotterdam, whereas the autocorrelation statistics (4–6, 8, 9, and 10) are consistently higher for AIDS. However, the differences are not very large.

Something must also be said about possible small-sample bias. Simulation studies in Meisner (1979), Bera *et al.* (1981), and Bewley (1986) have amply illustrated that the asymptotic LR test for symmetry in the static model is biased towards rejection. In the case of homogeneity this is also known from the theoretical arguments in Anderson (1958); and the more recent results in Rothenberg (1984) indicate that for Tests 1–3, 7, 11, and 12, the LR statistic is, to order T^{-1}, a multiple of chi-square under the null, and a simple average of the Wald and LM statistics. Byron and Rosalsky (1985) explicitly compute Edgeworth corrections for the statistics of Test no. 1 and report that this involves rather extensive computational effort. This is even more true of simulation-based corrections.

Heuristic, but less burdensome, alternatives are suggested by Böhm *et al.* (1980), who recommend multiplying both LR and LM by $(T-k)/T$, where k is the number of regressors per equation; and by Italianer (1985), who recommends a similar correction for the LR test. We agree with Böhm *et al.* that the correction is warranted for Tests 1 and 2. However, its properties in the remaining cases have not been thoroughly investigated. In view of the large differences between LR and LM, it would clearly be misleading to apply the same correction factor uniformly to all the statistics in Table IV. For Tests 1 and 2, the correction does not pull any of the statistics out of the critical region. We therefore leave out this issue as unresolved and make no small sample adjustments.

Space considerations do not allow us to report the full sets of regression and autocorrelation coefficients. The matrices R_1 and R_4 do not appear to be diagonal: for the homogeneous Rotterdam system, R_1 includes $n_{1,D} = 4$ significant diagonal elements, and $n_{1,O} = 3$ significant off-diagonal elements; and R_4 includes $n_{4,D} = 2$ significant diagonal elements, and $n_{4,O} = 8$ significant off-diagonal elements (we count as significant those coefficients with absolute values larger than three times the estimated standard error). The corresponding figures for the AIDS model are $n_{1,D} = 4$, $n_{1,O} = 2$, $n_{4,D} = 2$, and $n_{4,O} = 9$. Using the same significance criterion, the symmetric autocorrelated Rotterdam model classifies as Hicksian substitutes 'Food' and 'Housing', 'Food' and 'Durables', 'Drink and Tobacco' and 'Other Services' 'Durables' and 'Other Goods', 'Durables' and 'Other Services', 'Vehicles' and 'Other Services'. There are no significant Hicksian complements. The most significant substitution relation is between 'Food' and 'Durables', with a t-ratio of 7·28.

5. CONCLUSIONS

This paper has attempted to provide a methodology for jointly testing autocorrelation and regularity in allocation systems. It has been argued that both issues cannot be treated separately. Most of the tests that we propose require only the estimation of a linear allocation system, and hence do not involve an inordinate amount of computational effort. By contrast, the estimation of a large autocorrelated allocation system is quite costly.

We depart from usual practice in estimating equations for both durable and non-durable commodities, using data on expenditures and purchase price indices. Nevertheless, our results indicate that the parameters of all equations can be plausibly estimated, provided that autocorrelation and unit roots are taken into account: the estimated short-run elasticities in the last columns of Table II have a clear economic interpretation and are statistically consistent with homogeneity.

The estimated elasticities in the correlated and uncorrelated versions of the Rotterdam and differential AIDS systems are reasonably close. This is perhaps not too surprising, since neglecting autocorrelation produces consistent, albeit asymptotically inefficient, estimated coefficients. Nevertheless, as shown in Table IV, the inconsistency of the estimated variances has serious consequences on the various test statistics for regularity.

There is weak evidence against symmetry in Tests 7 and 11, where the classical conflict between LR and LM tests emerges (LR rejects but LM does not). This conflict might be resolved with new theoretical results on the small sample distributions of the statistics. Clearly, this is a prime topic for further research.

Our approach of estimating differential autocorrelated allocation systems has the advantage of being much more parsimonious in the number of parameters than the general dynamic approach. Yet another possibility is to reduce autocorrelation by augmenting the list of regressors, i.e. introducing conditioning variables or price expectations in equations (37)–(39) (such an attempt is made in Deschamps, 1992). The various tests of Table I should enable the investigator to ascertain whether the added explanatory variables fully account for the observed dynamic behaviour of the explained variables, and should therefore be quite useful in this context.

ACKNOWLEDGEMENTS

This work was undertaken while the author was Faculty Research Fellow at the CentER for Economic Research, Tilburg University, whose financial support is gratefully acknowledged. He benefited from helpful comments by A. P. Barten, F. Carlevaro, A. Trognon, and anonymous referees. Previous versions of this paper were presented at CentER, at the University of Geneva, at the First European Conference on Structure and Dynamics in Econometrics (Amsterdam, 1990), and at the Conference on Recent Advances in Consumption Analysis (Paris, October 1991). Any errors are the author's.

REFERENCES

Aitchison, J., and S. D. Silvey (1960), 'Maximum likelihood estimation procedures and associated tests of significance', *Journal of the Royal Statistical Society*, Series B, **22**, 154–171.

Anderson, T. W. (1958), *An Introduction to Multivariate Statistical Analysis*, Wiley, New York.

Anderson, G., and R. Blundell (1982), 'Estimation and hypothesis testing in dynamic singular equation systems', *Econometrica*, **50**, 1559–1571.

Anderson, G., and R. Blundell (1983), 'Testing restrictions in a flexible dynamic demand system: an application to consumers' expenditure in Canada', *Review of Economic Studies*, **50**, 397–410.

Anderson, G., and R. Blundell (1984), 'Consumer non-durables in the U.K.: a dynamic demand system', *Economic Journal*, **94** (supplement), 35–44.

Balestra, P. (1976), *La Dérivation Matricielle*, Sirey, Paris.

Barten, A. P. (1969), 'Maximum likelihood estimation of a complete system of demand equations', *European Economic Review*, **1**, 7–73.

Bera, A. K., R. P. Byron, and C. M. Jarque (1981), 'Further evidence on asymptotic tests for homogeneity and symmetry in large demand systems', *Economics Letters*, **8**, 101–105.

Berndt, E. R., and N. E. Savin (1975), 'Estimation and hypothesis testing in singular equation systems with autoregressive disturbances', *Econometrica*, **43**, 937–957.

Berndt, E. R., and N. E. Savin (1977), 'Conflict among criteria for testing hypotheses in the multivariate linear regression model', *Econometrica*, **45**, 1263–1277.

Bewley, R. A. (1986), *Allocation Models: Specification, Estimation and Applications*, Ballinger, Cambridge, MA.

Bewley, R. A., and G. Elliott (1992), 'Accounting for non-stationarity in demand systems', in R. Bewley and Tran Van Hoa (eds), *Contributions to Consumer Demand and Econometrics: Essays in Honour of Henri Theil*, Macmillan, London.

Böhm, B., B. Rieder, and G. Tintner (1980), 'A system of demand equations for Austria', *Empirical Economics*, **5**, 129–142.

Breusch, T. S. (1979), 'Conflict among criteria for testing hypotheses: extensions and comments', *Econometrica*, **47**, 203–207.

Breusch, T. S., and A. R. Pagan (1980), 'The Lagrange multiplier test and its applications to model specification in econometrics', *Review of Economic Studies*, **47**, 239–253.

Byron, R. and M. C. Rosalsky (1985), 'Hypothesis testing in demand systems: some examples of size corrections using Edgeworth approximations', *Econometric Theory*, **1**, 403–408.

Davidson, R., and J. G. MacKinnon (1980), 'Estimating the covariance matrix for regression models with AR(1) errors and lagged dependent variables', *Economics Letters*, **6**, 119–123.

Deaton, A. S., and J. Muellbauer (1980a), 'An almost ideal demand system', *American Economic Review*, **70**, 312–326.

Deaton, A. S., and J. Muellbauer (1980b), *Economics and Consumer Behaviour*, Cambridge University Press, Cambridge.

Deschamps, P. J. (1988), 'A note on the maximum likelihood estimation of allocation systems', *Computational Statistics and Data Analysis*, **6**, 109–112.

Deschamps, P. J. (1992), 'Expectations and intertemporal separability in an empirical model of consumption and investment under uncertainty', *Empirical Economics*, **17**, 419–450.

Godfrey, L. G. (1978), 'Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables', *Econometrica*, **46**, 1293–1301.

Granger, C. W. J., and P. Newbold (1974), 'Spurious regressions in econometrics', *Journal of Econometrics*, **2**, 111–120.

Hendry, D. F. (1971), 'Maximum likelihood estimation of systems of simultaneous regression equations with errors generated by a vector autoregressive process', *International Economic Review*, **12**, 257–272.

Hillier, G. H. (1991), 'On multiple diagnostic procedures for the linear model', *Journal of Econometrics*, **47**, 47–66.

Italianer, A. (1985), 'A small sample correction for the likelihood ratio test', *Economics Letters*, **19**, 315–317.

Laitinen, K. (1978), 'Why is demand homogeneity so often rejected?' *Economics Letters*, **1**, 187–191.

Lau, L. J. (1978), 'A note on the compatibility of a system of difference equations and a time-independent linear equation', *Economics Letters*, **1**, 243–247.

Meisner, J. F. (1979), 'The sad fate of the asymptotic Slutsky symmetry test for large systems', *Economics Letters*, **2**, 231–233.

Muellbauer, J. (1981), 'Testing neoclassical models of the demand for consumer durables', in A. Deaton (ed.), *Essays in the Theory and Measurement of Consumer Behaviour in Honour of Sir Richard Stone*, Cambridge University Press, Cambridge.

Nakamura, S. (1986), 'A flexible dynamic model of multiproduct technology for the West German economy', *Journal of Applied Econometrics*, **1**, 333–344.

Quandt, R. E. (1983), 'Computational problems and methods', in Z. Griliches and M. D. Intriligator (eds), *Handbook of Econometrics*, Vol. I, North-Holland, Amsterdam.

Rao, C. R. (1948), 'Large sample tests of statistical hypotheses concerning several parameters with applications to problems of estimation', *Proceedings of the Cambridge Philosophical Society*, **44**, 50–57.

Rothenberg, T. J. (1984), 'Hypothesis testing in linear models when the error covariance matrix is nonscalar', *Econometrica*, **52**, 827–842.

Savin, N. E. (1976), 'Conflict among testing procedures in a linear regression model with autoregressive disturbances', *Econometrica*, **44**, 1303–1315.

Theil, H. (1975), *Theory and Measurement of Consumer Demand*, Vols I and II, North-Holland, Amsterdam.

Veall, M. R., and K. F. Zimmermann (1986), 'A monthly dynamic consumer expenditure system for Germany with different kinds of households', *Review of Economics and Statistics*, **68**, 256–264.

White, H. (1982), 'Maximum likelihood estimation of misspecified models', *Econometrica*, **50**, 1–26.

Reprint Series, CentER, Tilburg University, The Netherlands:

No. 1 G. Marini and F. van der Ploeg, Monetary and fiscal policy in an optimising model with capital accumulation and finite lives, *The Economic Journal*, vol. 98, no. 392, 1988, pp. 772 - 786.

No. 2 F. van der Ploeg, International policy coordination in interdependent monetary economies, *Journal of International Economics*, vol. 25, 1988, pp. 1 - 23.

No. 3 A.P. Barten, The history of Dutch macroeconomic modelling (1936-1986), in W. Driehuis, M.M.G. Fase and H. den Hartog (eds.), *Challenges for Macroeconomic Modelling*, Contributions to Economic Analysis 178, Amsterdam: North-Holland, 1988, pp. 39 - 88.

No. 4 F. van der Ploeg, Disposable income, unemployment, inflation and state spending in a dynamic political-economic model, *Public Choice*, vol. 60, 1989, pp. 211 - 239.

No. 5 Th. ten Raa and F. van der Ploeg, A statistical approach to the problem of negatives in input-output analysis, *Economic Modelling*, vol. 6, no. 1, 1989, pp. 2 - 19.

No. 6 E. van Damme, Renegotiation-proof equilibria in repeated prisoners' dilemma, *Journal of Economic Theory*, vol. 47, no. 1, 1989, pp. 206 - 217.

No. 7 C. Mulder and F. van der Ploeg, Trade unions, investment and employment in a small open economy: a Dutch perspective, in J. Muysken and C. de Neubourg (eds.), *Unemployment in Europe*, London: The Macmillan Press Ltd, 1989, pp. 200 - 229.

No. 8 Th. van de Klundert and F. van der Ploeg, Wage rigidity and capital mobility in an optimizing model of a small open economy, *De Economist*, vol. 137, nr. 1, 1989, pp. 47 - 75.

No. 9 G. Dhaene and A.P. Barten, When it all began: the 1936 Tinbergen model revisited, *Economic Modelling*, vol. 6, no. 2, 1989, pp. 203 - 219.

No. 10 F. van der Ploeg and A.J. de Zeeuw, Conflict over arms accumulation in market and command economies, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amster- dam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 91 - 119.

No. 11 J. Driffill, Macroeconomic policy games with incomplete information: some extensions, in F. van der Ploeg and A.J. de Zeeuw (eds.), *Dynamic Policy Games in Economics*, Contributions to Economic Analysis 181, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 289 - 322.

No. 12 F. van der Ploeg, Towards monetary integration in Europe, in P. De Grauwe et al., *De Europese Monetaire Integratie: vier visies*, Wetenschappelijke Raad voor het Regeringsbeleid V 66, 's-Gravenhage: SDU uitgeverij, 1989, pp. 81 - 106.

No. 13 R.J.M. Alessie and A. Kapteyn, Consumption, savings and demography, in A. Wenig, K.F. Zimmermann (eds.), *Demographic Change and Economic Development*, Berlin/Heidelberg: Springer-Verlag, 1989, pp. 272 - 305.

No. 14 A. Hoque, J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model, *Journal of Econometrics*, vol. 39, no. 3, 1988, pp. 327 - 346.

No. 15 R. Alessie, A. Kapteyn and B. Melenberg, The effects of liquidity constraints on consumption: estimation from household panel data, *European Economic Review*, vol. 33, no. 2/3, 1989, pp. 547 - 555.

No. 16 A. Holly and J.R. Magnus, A note on instrumental variables and maximum likelihood estimation procedures, *Annales d'Économie et de Statistique*, no. 10, April-June, 1988, pp. 121 - 138.

No. 17 P. ten Hacken, A. Kapteyn and I. Woittiez, Unemployment benefits and the labor market, a micro/macro approach, in B.A. Gustafsson and N. Anders Klevmarken (eds.), *The Political Economy of Social Security*, Contributions to Economic Analysis 179, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1989, pp. 143 - 164.

No. 18 T. Wansbeek and A. Kapteyn, Estimation of the error-components model with incomplete panels, *Journal of Econometrics*, vol. 41, no. 3, 1989, pp. 341 - 361.

No. 19 A. Kapteyn, P. Kooreman and R. Willemse, Some methodological issues in the implementation of subjective poverty definitions, *The Journal of Human Resources*, vol. 23, no. 2, 1988, pp. 222 - 242.

No. 20 Th. van de Klundert and F. van der Ploeg, Fiscal policy and finite lives in interdependent economies with real and nominal wage rigidity, *Oxford Economic Papers*, vol. 41, no. 3, 1989, pp. 459 - 489.

No. 21 J.R. Magnus and B. Pesaran, The exact multi-period mean-square forecast error for the first-order autoregressive model with an intercept, *Journal of Econometrics*, vol. 42, no. 2, 1989, pp. 157 - 179.

No. 22 F. van der Ploeg, Two essays on political economy: (i) The political economy of overvaluation, *The Economic Journal*, vol. 99, no. 397, 1989, pp. 850 - 855; (ii) Election outcomes and the stockmarket, *European Journal of Political Economy*, vol. 5, no. 1, 1989, pp. 21 - 30.

No. 23 J.R. Magnus and A.D. Woodland, On the maximum likelihood estimation of multivariate regression models containing serially correlated error components, *International Economic Review*, vol. 29, no. 4, 1988, pp. 707 - 725.

No. 24 A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for stationary point problems on polytopes, *Mathematics of Operations Research*, vol. 14, no. 3, 1989, pp. 383 - 399.

No. 25 E. van Damme, Stable equilibria and forward induction, *Journal of Economic Theory*, vol. 48, no. 2, 1989, pp. 476 - 496.

No. 26 A.P. Barten and L.J. Bettendorf, Price formation of fish: An application of an inverse demand system, *European Economic Review*, vol. 33, no. 8, 1989, pp. 1509 - 1525.

No. 27 G. Noldeke and E. van Damme, Signalling in a dynamic labour market, *Review of Economic Studies*, vol. 57 (1), no. 189, 1990, pp. 1 - 23.

No. 28 P. Kop Jansen and Th. ten Raa, The choice of model in the construction of input-output coefficients matrices, *International Economic Review*, vol. 31, no. 1, 1990, pp. 213 - 227.

No. 29 F. van der Ploeg and A.J. de Zeeuw, Perfect equilibrium in a model of competitive arms accumulation, *International Economic Review*, vol. 31, no. 1, 1990, pp. 131 - 146.

No. 30 J.R. Magnus and A.D. Woodland, Separability and aggregation, *Economica*, vol. 57, no. 226, 1990, pp. 239 - 247.

No. 31 F. van der Ploeg, International interdependence and policy coordination in economies with real and nominal wage rigidity, *Greek Economic Review*, vol. 10, no. 1, June 1988, pp. 1 - 48.

No. 32 E. van Damme, Signaling and forward induction in a market entry context, *Operations Research Proceedings 1989*, Berlin-Heidelberg: Springer-Verlag, 1990, pp. 45 - 59.

No. 33 A.P. Barten, Toward a levels version of the Rotterdam and related demand systems, *Contributions to Operations Research and Economics*, Cambridge: MIT Press, 1989, pp. 441 - 465.

No. 34 F. van der Ploeg, International coordination of monetary policies under alternative exchange-rate regimes, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 91 - 121.

No. 35 Th. van de Klundert, On socioeconomic causes of 'wait unemployment', *European Economic Review*, vol. 34, no. 5, 1990, pp. 1011 - 1022.

No. 36 R.J.M. Alessie, A. Kapteyn, J.B. van Lochem and T.J. Wansbeek, Individual effects in utility consistent models of demand, in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1990, pp. 253 - 278.

No. 37 F. van der Ploeg, Capital accumulation, inflation and long-run conflict in international objectives, *Oxford Economic Papers*, vol. 42, no. 3, 1990, pp. 501 - 525.

No. 38 Th. Nijman and F. Palm, Parameter identification in ARMA Processes in the presence of regular but incomplete sampling, *Journal of Time Series Analysis*, vol. 11, no. 3, 1990, pp. 239 - 248.

No. 39 Th. van de Klundert, Wage differentials and employment in a two-sector model with a dual labour market, *Metroeconomica*, vol. 40, no. 3, 1989, pp. 235 - 256.

No. 40 Th. Nijman and M.F.J. Steel, Exclusion restrictions in instrumental variables equations, *Econometric Reviews*, vol. 9, no. 1, 1990, pp. 37 - 55.

No. 41 A. van Soest, I. Woittiez and A. Kapteyn, Labor supply, income taxes, and hours restrictions in the Netherlands, *Journal of Human Resources*, vol. 25, no. 3, 1990, pp. 517 - 558.

No. 42 Th.C.M.J. van de Klundert and A.B.T.M. van Schaik, Unemployment persistence and loss of productive capacity: a Keynesian approach, *Journal of Macro- economics*, vol. 12, no. 3, 1990, pp. 363 - 380.

No. 43 Th. Nijman and M. Verbeek, Estimation of time-dependent parameters in linear models using cross-sections, panels, or both, *Journal of Econometrics*, vol. 46, no. 3, 1990, pp. 333 - 346.

No. 44 E. van Damme, R. Selten and E. Winter, Alternating bid bargaining with a smallest money unit, *Games and Economic Behavior*, vol. 2, no. 2, 1990, pp. 188 - 201.

No. 45 C. Dang, The D_1-triangulation of $\mathbb{R}^n$ for simplicial algorithms for computing solutions of nonlinear equations, *Mathematics of Operations Research*, vol. 16, no. 1, 1991, pp. 148 - 161.

No. 46 Th. Nijman and F. Palm, Predictive accuracy gain from disaggregate sampling in ARIMA models, *Journal of Business & Economic Statistics*, vol. 8, no. 4, 1990, pp. 405 - 415.

No. 47 J.R. Magnus, On certain moments relating to ratios of quadratic forms in normal variables: further results, *Sankhya: The Indian Journal of Statistics*, vol. 52, series B, part. 1, 1990, pp. 1 - 13.

No. 48 M.F.J. Steel, A Bayesian analysis of simultaneous equation models by combining recursive analytical and numerical approaches, *Journal of Econometrics*, vol. 48, no. 1/2, 1991, pp. 83 - 117.

No. 49 F. van der Ploeg and C. Withagen, Pollution control and the ramsey problem, *Environmental and Resource Economics*, vol. 1, no. 2, 1991, pp. 215 - 236.

No. 50 F. van der Ploeg, Money and capital in interdependent economies with overlapping generations, *Economica*, vol. 58, no. 230, 1991, pp. 233 - 256.

No. 51 A. Kapteyn and A. de Zeeuw, Changing incentives for economic research in the Netherlands, *European Economic Review*, vol. 35, no. 2/3, 1991, pp. 603 - 611.

No. 52 C.G. de Vries, On the relation between GARCH and stable processes, *Journal of Econometrics*, vol. 48, no. 3, 1991, pp. 313 - 324.

No. 53 R. Alessie and A. Kapteyn, Habit formation, interdependent preferences and demographic effects in the almost ideal demand system, The Economic Journal, vol. 101, no. 406, 1991, pp. 404 - 419.

No. 54 W. van Groenendaal and A. de Zeeuw, Control, coordination and conflict on international commodity markets, Economic Modelling, vol. 8, no. 1, 1991, pp. 90 - 101.

No. 55 F. van der Ploeg and A.J. Markink, Dynamic policy in linear models with rational expectations of future events: A computer package, Computer Science in Economics and Management, vol. 4, no. 3, 1991, pp. 175 - 199.

No. 56 H.A. Keuzenkamp and F. van der Ploeg, Savings, investment, government finance, and the current account: The Dutch experience, in G. Alogoskoufis, L. Papademos and R. Portes (eds.), External Constraints on Macroeconomic Policy: The European Experience, Cambridge: Cambridge University Press, 1991, pp. 219 - 263.

No. 57 Th. Nijman, M. Verbeek and A. van Soest, The efficiency of rotating-panel designs in an analysis-of-variance model, Journal of Econometrics, vol. 49, no. 3, 1991, pp. 373 - 399.

No. 58 M.F.J. Steel and J.-F. Richard, Bayesian multivariate exogeneity analysis - an application to a UK money demand equation, Journal of Econometrics, vol. 49, no. 1/2, 1991, pp. 239 - 274.

No. 59 Th. Nijman and F. Palm, Generalized least squares estimation of linear models containing rational future expectations, International Economic Review, vol. 32, no. 2, 1991, pp. 383 - 389.

No. 60 E. van Damme, Equilibrium selection in 2 x 2 games, Revista Espanola de Economia, vol. 8, no. 1, 1991, pp. 37 - 52.

No. 61 E. Bennett and E. van Damme, Demand commitment bargaining: the case of apex games, in R. Selten (ed.), Game Equilibrium Models III - Strategic Bargaining, Berlin: Springer-Verlag, 1991, pp. 118 - 140.

No. 62 W. Güth and E. van Damme, Gorby games - a game theoretic analysis of disarmament campaigns and the defense efficiency - hypothesis -, in R. Avenhaus, H. Karkar and M. Rudnianski (eds.), Defense Decision Making - Analytical Support and Crisis Management, Berlin: Springer-Verlag, 1991, pp. 215 - 240.

No. 63 A. Roell, Dual-capacity trading and the quality of the market, *Journal of Financial Intermediation*, vol. 1, no. 2, 1990, pp. 105 - 124.

No. 64 Y. Dai, G. van der Laan, A.J.J. Talman and Y. Yamamoto, A simplicial algorithm for the nonlinear stationary point problem on an unbounded polyhedron, *Siam Journal of Optimization*, vol. 1, no. 2, 1991, pp. 151 - 165.

No. 65 M. McAleer and C.R. McKenzie, Keynesian and new classical models of unemployment revisited, *The Economic Journal*, vol. 101, no. 406, 1991, pp. 359 - 381.

No. 66 A.J.J. Talman, General equilibrium programming, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 387 - 397.

No. 67 J.R. Magnus and B. Pesaran, The bias of forecasts from a first-order autoregression, *Econometric Theory*, vol. 7, no. 2, 1991, pp. 222 - 235.

No. 68 F. van der Ploeg, Macroeconomic policy coordination issues during the various phases of economic and monetary integration in Europe, *European Economy - The Economics of EMU*, Commission of the European Communities, special edition no. 1, 1991, pp. 136 - 164.

No. 69 H. Keuzenkamp, A precursor to Muth: Tinbergen's 1932 model of rational expectations, *The Economic Journal*, vol. 101, no. 408, 1991, pp. 1245 - 1253.

No. 70 L. Zou, The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect, *Journal of Public Economics*, vol. 46, no. 1, 1991, pp. 51 - 89.

No. 71 E. Bomhoff, Between price reform and privatization: Eastern Europe in transition, *Finanzmarkt und Portfolio Management*, vol. 5, no. 3, 1991, pp. 241 - 251.

No. 72 E. Bomhoff, Stability of velocity in the major industrial countries: a Kalman filter approach, *International Monetary Fund Staff Papers*, vol. 38, no. 3, 1991, pp. 626 - 642.

No. 73 E. Bomhoff, Currency convertibility: when and how? A contribution to the Bulgarian debate, *Kredit und Kapital*, vol. 24, no. 3, 1991, pp. 412 - 431.

No. 74 H. Keuzenkamp and F. van der Ploeg, Perceived constraints for Dutch unemployment policy, in C. de Neubourg (ed.), *The Art of Full Employment - Unemployment Policy in Open Economies*, Contributions to Economic Analysis 203, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1991, pp. 7 - 37.

No. 75 H. Peters and E. van Damme, Characterizing the Nash and Raiffa bargaining solutions by disagreement point axions, *Mathematics of Operations Research*, vol. 16, no. 3, 1991, pp. 447 - 461.

No. 76 P.J. Deschamps, On the estimated variances of regression coefficients in misspecified error components models, *Econometric Theory*, vol. 7, no. 3, 1991, pp. 369 - 384.

No. 77 A. de Zeeuw, Note on 'Nash and Stackelberg solutions in a differential game model of capitalism', *Journal of Economic Dynamics and Control*, vol. 16, no. 1, 1992, pp. 139 - 145.

No. 78 J.R. Magnus, On the fundamental bordered matrix of linear estimation, in F. van der Ploeg (ed.), *Advanced Lectures in Quantitative Economics*, London-Orlando: Academic Press Ltd., 1990, pp. 583 - 604.

No. 79 F. van der Ploeg and A. de Zeeuw, A differential game of international pollution control, *Systems and Control Letters*, vol. 17, no. 6, 1991, pp. 409 - 414.

No. 80 Th. Nijman and M. Verbeek, The optimal choice of controls and pre-experimen- tal observations, *Journal of Econometrics*, vol. 51, no. 1/2, 1992, pp. 183 - 189.

No. 81 M. Verbeek and Th. Nijman, Can cohort data be treated as genuine panel data?, *Empirical Economics*, vol. 17, no. 1, 1992, pp. 9 - 23.

No. 82 E. van Damme and W. Güth, Equilibrium selection in the Spence signaling game, in R. Selten (ed.), *Game Equilibrium Models II - Methods, Morals, and Markets*, Berlin: Springer-Verlag, 1991, pp. 263 - 288.

No. 83 R.P. Gilles and P.H.M. Ruys, Characterization of economic agents in arbitrary communication structures, *Nieuw Archief voor Wiskunde*, vol. 8, no. 3, 1990, pp. 325 - 345.

No. 84 A. de Zeeuw and F. van der Ploeg, Difference games and policy evaluation: a conceptual framework, *Oxford Economic Papers*, vol. 43, no. 4, 1991, pp. 612 - 636.

No. 85 E. van Damme, Fair division under asymmetric information, in R. Selten (ed.), *Rational Interaction - Essays in Honor of John C. Harsanyi*, Berlin/Heidelberg: Springer-Verlag, 1992, pp. 121 - 144.

No. 86 F. de Jong, A. Kemna and T. Kloek, A contribution to event study methodology with an application to the Dutch stock market, *Journal of Banking and Finance*, vol. 16, no. 1, 1992, pp. 11 - 36.

No. 87 A.P. Barten, The estimation of mixed demand systems, in R. Bewley and T. Van Hoa (eds.), *Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 31 - 57.

No. 88 T. Wansbeek and A. Kapteyn, Simple estimators for dynamic panel data models with errors in variables, in R. Bewley and T. Van Hoa (eds.), *Contributions to Consumer Demand and Econometrics, Essays in Honour of Henri Theil*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 238 - 251.

No. 89 S. Chib, J. Osiewalski and M. Steel, Posterior inference on the degrees of freedom parameter in multivariate-*t* regression models, *Economics Letters*, vol. 37, no. 4, 1991, pp. 391 - 397.

No. 90 H. Peters and P. Wakker, Independence of irrelevant alternatives and revealed group preferences, *Econometrica*, vol. 59, no. 6, 1991, pp. 1787 - 1801.

No. 91 G. Alogoskoufis and F. van der Ploeg, On budgetary policies, growth, and external deficits in an interdependent world, *Journal of the Japanese and International Economies*, vol. 5, no. 4, 1991, pp. 305 - 324.

No. 92 R.P. Gilles, G. Owen and R. van den Brink, Games with permission structures: The conjunctive approach, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 277 - 293.

No. 93 J.A.M. Potters, I.J. Curiel and S.H. Tijs, Traveling salesman games, *Mathematical Programming*, vol. 53, no. 2, 1992, pp. 199 - 211.

No. 94 A.P. Jurg, M.J.M. Jansen, J.A.M. Potters and S.H. Tijs, A symmetrization for finite two-person games, *Zeitschrift für Operations Research - Methods and Models of Operations Research*, vol. 36, no. 2, 1992, pp. 111 - 123.

No. 95 A. van den Nouweland, P. Borm and S. Tijs, Allocation rules for hypergraph communication situations, *International Journal of Game Theory*, vol. 20, no. 3, 1992, pp. 255 - 268.

No. 96 E.J. Bomhoff, Monetary reform in Eastern Europe, *European Economic Review*, vol. 36, no. 2/3, 1992, pp. 454 - 458.

No. 97 F. van der Ploeg and A. de Zeeuw, International aspects of pollution control, *Environmental and Resource Economics*, vol. 2, no. 2, 1992, pp. 117 - 139.

No. 98 P.E.M. Borm and S.H. Tijs, Strategic claim games corresponding to an NTU-game, *Games and Economic Behavior*, vol. 4, no. 1, 1992, pp. 58 - 71.

No. 99 A. van Soest and P. Kooreman, Coherency of the indirect translog demand system with binding nonnegativity constraints, *Journal of Econometrics*, vol. 44, no. 3, 1990, pp. 391 - 400.

No. 100 Th. ten Raa and E.N. Wolff, Secondary products and the measurement of productivity growth, *Regional Science and Urban Economics*, vol. 21, no. 4, 1991, pp. 581 - 615.

No. 101 P. Kooreman and A. Kapteyn, On the empirical implementation of some game theoretic models of household labor supply, *The Journal of Human Resources*, vol. 25, no. 4, 1990, pp. 584 - 598.

No. 102 H. Bester, Bertrand equilibrium in a differentiated duopoly, *International Economic Review*, vol. 33, no. 2, 1992, pp. 433 - 448.

No. 103 J.A.M. Potters and S.H. Tijs, The nucleolus of a matrix game and other nucleoli, *Mathematics of Operations Research*, vol. 17, no. 1, 1992, pp. 164 - 174.

No. 104 A. Kapteyn, P. Kooreman and A. van Soest, Quantity rationing and concavity in a flexible household labor supply model, *Review of Economics and Statistics*, vol. 72, no. 1, 1990, pp. 55 - 62.

No. 105 A. Kapteyn and P. Kooreman, Household labor supply: What kind of data can tell us how many decision makers there are?, *European Economic Review*, vol. 36, no. 2/3, 1992, pp. 365 - 371.

No. 106 Th. van de Klundert and S. Smulders, Reconstructing growth theory: A survey, *De Economist*, vol. 140, no. 2, 1992, pp. 177 - 203.

No. 107 N. Rankin, Imperfect competition, expectations and the multiple effects of monetary growth, *The Economic Journal*, vol. 102, no. 413, 1992, pp. 743 - 753.

No. 108 J. Greenberg, On the sensitivity of von Neumann and Morgenstern abstract stable sets: The stable and the individual stable bargaining set, *International Journal of Game Theory*, vol. 21, no. 1, 1992, pp. 41 - 55.

No. 109 S. van Wijnbergen, Trade reform, policy uncertainty, and the current account: A non-expected-utility approach, *American Economic Review*, vol. 82, no. 3, 1992, pp. 626 - 633.

No. 110 M. Verbeek and Th. Nijman, Testing for selectivity bias in panel data models, *International Economic Review*, vol. 33, no. 3, 1992, pp. 681 - 703.

No. 111 Th. Nijman and M. Verbeek, Nonresponse in panel data: The impact on estimates of a life cycle consumption function, *Journal of Applied Econometrics*, vol. 7, no. 3, 1992, pp. 243 - 257.

No. 112 I. Bomze and E. van Damme, A dynamical characterization of evolutionarily stable states, *Annals of Operations Research*, vol. 37, 1992, pp. 229 - 244.

No. 113 P.J. Deschamps, Expectations and intertemporal separability in an empirical model of consumption and investment under uncertainty, *Empirical Economics*, vol. 17, no. 3, 1992, pp. 419 - 450.

No. 114 K. Kamiya and D. Talman, Simplicial algorithm for computing a core element in a balanced game, *Journal of the Operations Research*, vol. 34, no. 2, 1991, pp. 222 - 228.

No. 115 G.W. Imbens, An efficient method of moments estimator for discrete choice models with choice-based sampling, *Econometrica*, vol. 60, no. 5, 1992, pp. 1187 -1214.

No. 116 P. Borm, On perfectness concepts for bimatrix games, *OR Spektrum*, vol. 14, no. 1, 1992, pp. 33 - 42.

No. 117 A.P. Jurg, I. Garcia Jurado and P.E.M. Borm, On modifications of the concepts of perfect and proper equilibria, *OR Spektrum*, vol. 14, no. 2, 1992, pp. 85 - 90.

No. 118 P. Borm, H. Keiding, R.P. McLean, S. Oortwijn and S. Tijs, The compromise value for NTU-games, *International Journal of Game Theory*, vol. 21, no. 2, 1992, pp. 175 - 189.

No. 119 M. Maschler, J.A.M. Potters and S.H. Tijs, The general nucleolus and the reduced game property, *International Journal of Game Theory*, vol. 21, no. 1, 1992, pp. 85 - 106.

No. 120 K. Wärneryd, Communication, correlation and symmetry in bargaining, *Economics Letters*, vol. 39, no. 3, 1992, pp. 295 - 300.

No. 121 M.R. Baye, D. Kovenock and C.G. de Vries, It takes two to tango: equilibria in a model of sales, *Games and Economic Behavior*, vol. 4, no. 4, 1992, pp. 493 - 510.

No. 122 M. Verbeek, Pseudo panel data, in L. Mátyás and P. Sevestre (eds.), *The Econometrics of Panel Data*, Dordrecht: Kluwer Academic Publishers, 1992, pp. 303 - 315.

No. 123 S. van Wijnbergen, Intertemporal speculation, shortages and the political economy of price reform, *The Economic Journal*, vol. 102, no. 415, 1992, pp. 1395 - 1406.

No. 124 M. Verbeek and Th. Nijman, Incomplete panels and selection bias, in L. Mátyás and P. Sevestre (eds.), *The Econometrics of Panel Data*, Dordrecht: Kluwer Academic Publishers, 1992, pp. 262 - 302.

No. 125 J.J. Sijben, Monetary policy in a game-theoretic framework, *Jahrbücher für Nationalökonomie und Statistik*, vol. 210, no. 3/4, 1992, pp. 233 - 253.

No. 126 H.A.A. Verbon and M.J.M. Verhoeven, Decision making on pension schemes under rational expectations, *Journal of Economics*, vol. 56, no. 1, 1992, pp. 71 - 97.

No. 127 L. Zou, Ownership structure and efficiency: An incentive mechanism approach, *Journal of Comparative Economics*, vol. 16, no. 3, 1993, pp. 399 - 431.

No. 128 C. Fershtman and A. de Zeeuw, Capital accumulation and entry deterrence: A clarifying note, in G. Feichtinger (ed.), *Dynamic Economic Models and Optimal Control*, Amsterdam: Elsevier Science Publishers B.V. (North-Holland), 1992, pp. 281 - 296.

No. 129 L. Bovenberg and C. Petersen, Public debt and pension policy, *Fiscal Studies*, vol. 13, no. 3, 1992, pp. 1 - 14.

No. 130 R. Gradus and A. de Zeeuw, An employment game between government and firms, *Optimal Control Applications & Methods*, vol. 13, no. 1, 1992, pp. 55 - 71.

No. 131 Th. Nijman and R. Beetsma, Empirical tests of a simple pricing model for sugar futures, *Annales d'Économie et de Statistique*, no. 24, 1991, pp. 121 - 131.

No. 132 F. Groot, C. Withagen and A. de Zeeuw, Note on the open-loop Von Stackelberg equilibrium in the Cartel versus Fringe model, *The Economic Journal*, vol. 102, no. 415, 1992, pp. 1478 - 1484.

No. 133 S. Eijffinger and N. Gruijters, On the effectiveness of daily intervention by the Deutsche Bundesbank and the Federal Reserve System in the US dollar - deutsche mark exchange market, in Baltensperger/Sinn (eds), *Exchange-Rate Regimes and Currency Unions*, Basingstoke: The Macmillan Press Ltd., 1992, pp. 131 - 156.

No. 135 A. K. Bera and S. Lee, Information matrix test, parameter heterogeneity and ARCH: a synthesis, *Review of Economic Studies*, 60, 1993, pp. 229 - 240.

No. 136 H. G. Bloemen and A. Kapteyn, The joint estimation of a non-linear labour supply function and a wage equation using simulated response probabilities, *Annales d'Économie et de Statistique*, No. 29, 1993, pp. 175 - 205.

No. 137 H. Bester, Bargaining versus price competition in markets with quality uncertainty, *The American Economic Review*, Vol. 83, No. 1, March 1993, pp. 278 - 288.

No. 138 K. Wärneryd, Anarchy, uncertainty, and the emergence of property rights, *Economics and Politics*, Vol. 5, No. 1, March 1993, pp. 1 - 14.

No. 139 A. L. Bovenberg and L.H. Goulder, Promoting investment under international capital mobility: an intertemporal general equilibrium analysis, *The Scandinavian Journal of Economics*, Vol. 95, No. 2, 1993, pp. 133 - 156.

No. 140 S. Eijffinger and E. Schaling, Central bank independence in twelve industrial countries, *Banca Nazionale del Lavoro Quarterly Review*, No. 184, March 1993, pp. 49 - 89.

No. 141 S. Eijffinger and A. van Rixtel, The Japanese financial system and monetary policy: a descriptive review, *Japan and the World Economy*, Vol. 4, No. 4, 1992, pp 291-309.

No. 142 A. L. Bovenberg, Investment-promoting policies in open economies: the importance of intergenerational and international distributional effects, *Journal of Public Economics*, Vol. 51, 1993, North Holland, pp. 3-54 .

No. 143 A. Özcam, G. Judge, A Bera and T. Yancey, The risk properties of a pre-test estimator for Zellner's seemingly unrelated regression model, *Journal of Quantitative Economics*, Vol. 9, No. 1, January 1993, pp. 41-52.

No. 144 F. C. Drost and T. E. Nijman, Temporal aggregation of garch processes, *Econometrica*, Vol. 61, No. 4, July 1993, pp. 909-927.

No. 145 J. J. G. Lemmen and S.C.W. Eijffinger, The degree of financial integration in the European Community, *De Economist*, Vol. 141, No. 2, 1993, pp. 189-213.

No. 146 R. Sarin and P. Wakker, A simple axiomatization of nonadditive expected utility, *Econometrica*, Vol. 60, No. 6, November 1992, pp. 1255-1272.

No. 147 S. Muto, On licensing policies in bertrand competition, *Games and Economic Behaviour*, 5, 1993, pp. 257-267.

No. 148 M. Verbeek and T. Nijman, Minimum MSE estimation of a regression model with fixed effects from a series of cross-sections, *Journal of Econometrics*, 59, 1993, pp. 125-136.

No. 149 R. de Groof and M. van Tuijl, Financial integration and fiscal policy in interdependent two-sector economies with real and nominal wage rigidity, *European Journal of Political Economy*, Vol. 9, 1993, North Holland, pp. 209-232.

No. 150 A. van Soest, A. Kapteyn and P. Kooreman, Coherency and regularity of demand systems with equality and inequality constraints, *Journal of Econometrics*, Vol. 57, 1993, North- Holland, pp. 161-188.

No. 151 W. Härdle and A.B. Tsybakov, How sensitive are average derivatives?, *Journal of Econometrics*, Vol. 58, 1993, North-Holland, pp. 31-48.

No. 152 H. Bester, The role of collateral in a model of debt renegotiation, *Journal of Money, Credit and Banking*, Vol. 26, No. 1, February 1994, pp. 72-86.

No. 153 J. J. Sijben, Credit markets, financial fragility, and the real economy, *Kredit und Kapital*, Vol. 26, No. 4, 1993, pp. 481-515.

No. 154 Y. Dai and D. Talman, Linear stationary point problems on unbounded polyhedra, *Mathematics of Operations Research*, Vol. 18, No. 3, August 1993, pp. 635-644.

No. 155 P. J. Deschamps, Joint tests for regularity and autocorrelation in allocation systems, *Journal of Applied Econometrics*, Vol. 8, 1993, pp. 195-211.

P.O. BOX 90153. 5000 LE TILBURG. THE NETHERLANDS